revisionvillage

Mathematics: Analysis & Approaches SL

1 Page Formula Sheet

Topic 1: Number & Algebra

1.2	
nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
Sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
1.3	
nth term of a geometric sequence	$u_n = u_1 r^{n-1}$
Sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$
1.8	
The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}$, $\lvert r \rvert < 1$
1.4	
Compound Interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ FV is future value, PV is present value, n is number of years, k is compounding periods per year, r% is nominal annual interest rate
1.5	
Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$, $a, b > 0, a \neq 1$
1.7	
Exponents and logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m\log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
Exponential and logarithmic functions	$a^x = e^{x \ln a}$; $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$
1.9	
Binomial theorem, $n \in \mathbb{N}$	$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n$ ${}^n\mathrm{C}_r = \frac{n!}{r!\,(n-r)!}$

Topic 2: Functions

2.1	
Equations of a straight line	$y = mx + c$ $ax + by + d = 0$ $y - y_1 = m(x - x_1)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2.6	
Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
2.7	
Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$
Discriminant	$\Delta = b^2 - 4ac$

Topic 3: Geometry & Trigonometry – Prior Learning

Area: parallelogram	$A = bh$, where b is base, h is height
Area: triangle	$A = \frac{1}{2}(bh)$, where b is base, h is height
Area: trapezoid	$A = \frac{1}{2}(a+b)h$, where a and b are the parallel sides, h is height
Area: circle	$A = \pi r^2$, where r is radius
Circumference: circle	$C = 2\pi r$, where r is radius
Volume: cuboid	$V = lwh$, l is length, w is width, h is height
Volume: cylinder	$V = \pi r^2 h$, where r is radius, h is height
Volume: prism	$V = Ah$, A is area of cross-section, h is height
Surface area: cylinder curve	$A = 2\pi rh$, where r is radius, h is height
Distance between 2 points $(x_1, y_1); (x_2, y_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of midpoint of $(x_1, y_1); (x_2, y_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Topic 3: Geometry & Trigonometry

3.1	
Distance between 2 points $(x_1, y_1, z_1); (x_2, y_2, z_2)$	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of midpoint of $(x_1, y_1, z_1); (x_2, y_2, z_2)$	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
Volume: right-pyramid	$V = \frac{1}{3}Ah$, where A is area of base, h is height
Volume: right-cone	$V = \frac{1}{3}\pi r^2 h$, where r is radius, h is height
Surface area: cone curve	$A = \pi rl$, where r is radius, l is slant height
Volume: sphere	$V = \frac{4}{3}\pi r^3$, where r is the radius
Surface area: sphere	$A = 4\pi r^2$, where r is the radius
3.2	
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$ $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
Area of a triangle	$A = \frac{1}{2}ab\sin C$
3.4	
Length of an arc	$l = r\theta$, where r is the radius, θ is the angle measured in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$, where r is the radius, θ is the angle measured in radians
3.5	
Identity for $\tan\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
3.6	
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
Double angle identities	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$

Topic 4: Statistics & Probability

4.2	
Interquartile range	$\mathrm{IQR} = Q_3 - Q_1$
4.3	
Mean, $\bar{x}$, of a data set	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ where $n = \sum_{i=1}^{k} f_i$
4.5	
Probability of an event A	$\mathrm{P}(A) = \frac{n(A)}{n(U)}$
Complementary events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
4.6	
Combined events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
Mutually exclusive events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
Conditional probability	$\mathrm{P}(A\lvert B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
Independent events	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$
4.7	
Expected value of a discrete random variable X	$\mathrm{E}(X) = \sum_{i=1}^{k} x_i \mathrm{P}(X = x_i)$
4.8	
Binomial distribution	$X \sim \mathrm{B}(n, p)$
Mean	$\mathrm{E}(X) = np$
Variance	$\mathrm{Var}(X) = np(1-p)$
4.12	
Standardized normal variable	$z = \frac{x - \mu}{\sigma}$

Topic 5: Calculus

5.3	
Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
5.6	
Standard derivatives	$f(x) = \sin x \Rightarrow f'(x) = \cos x$ $f(x) = \cos x \Rightarrow f'(x) = -\sin x$ $f(x) = e^x \Rightarrow f'(x) = e^x$ $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u)$, where $u = f(x) \Rightarrow$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$
Product rule	$y = uv \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
5.9	
Acceleration	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$
Distance travelled from t_1 to t_2	$\text{distance} = \int_{t_1}^{t_2} \lvert v(t) \rvert \,\mathrm{d}t$
Displacement from t_1 to t_2	$\text{displacement} = \int_{t_1}^{t_2} v(t)\,\mathrm{d}t$
5.5	
Integral of x^n	$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
Area between a curve $f(x)$ and the x-axis, $f(x) > 0$	$A = \int_a^b y \,\mathrm{d}x$
5.10	
Standard Integrals	$\int \frac{1}{x}\mathrm{d}x = \ln\lvert x \rvert + C$ $\int \sin x \,\mathrm{d}x = -\cos x + C$ $\int \cos x \,\mathrm{d}x = \sin x + C$ $\int e^x \,\mathrm{d}x = e^x + C$
5.11	
Area of region enclosed by a curve and x-axis	$A = \int_a^b \lvert y \rvert \,\mathrm{d}x$